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Fluid Mechanics [1]

MEC 121

1st Year- Mechanical Engineering

2nd Term-(2024)

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❖ HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES

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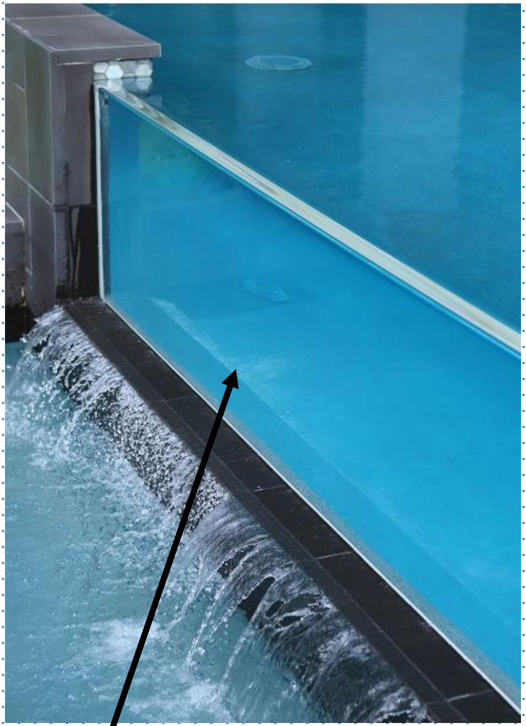
A plate (such as a gate valve in a dam, is subjected to fluid pressure distributed over its surface when exposed to a liquid.

➢ Examples on Hydrostatic Force

Inclined Surface



Vertical Surface **Horizontal Surface** **Curved Surface**

❖Do Car Doors Jam Underwater?

How To Survive A Submerging Car?

1- Think And Act Fast

2- Unbuckle And Break The Window

3- Let The Pressure Equalize

4- Swim Out And Get To The Surface

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❖ Objectives:

➢ ***we need to determine:***

- The **magnitude** of the force (**FR**)
- The **point of application $Y_{c.p}$**
- **Pressure Distribution on that surface**

➢ ***Method of Solution :***

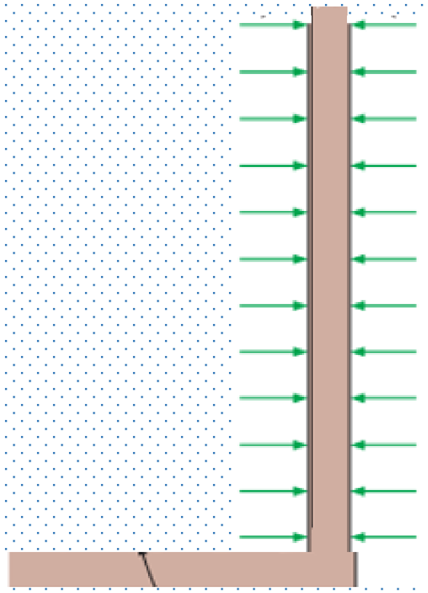
❖ **Exact Method**

❖ **Pressure Diagram Method** *(for rectangular Surface area)*

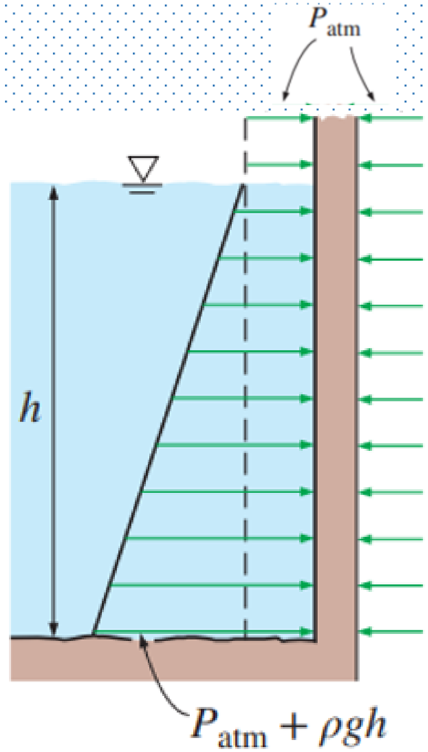
❖ Is it important to consider the atmospheric pressure in the hydrostatic force calculation?

- In most cases, the other side of the plate is open to the atmosphere (such as the dry side of a gate), and thus atmospheric pressure acts on both sides of the plate, yielding a zero resultant

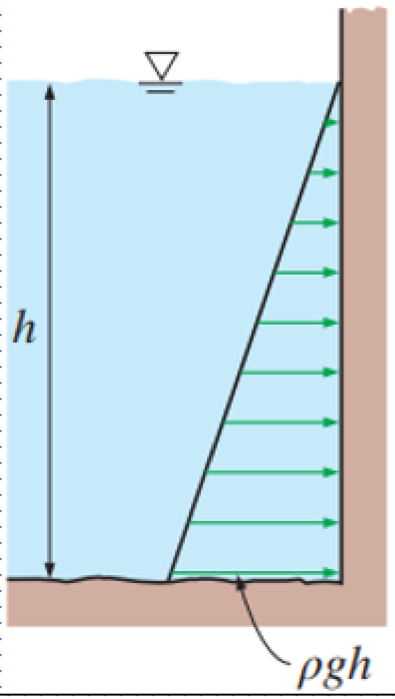
❑ P_{atm} Considered (No Fluid)



❑ P_{atm} Considered (with Fluid)



❑ P_{atm} Subtracted (Fluid Only)



❶ The **magnitude** of the force (F_R)

Find the Pressure at any point ------use the Integration to find the Force

The absolute pressure at any point on the plate is given by:

$$P = P_0 + \rho g(h) = P_0 + \rho g(y\sin\theta)$$ *where, from the drawing* $h = y\sin\theta$

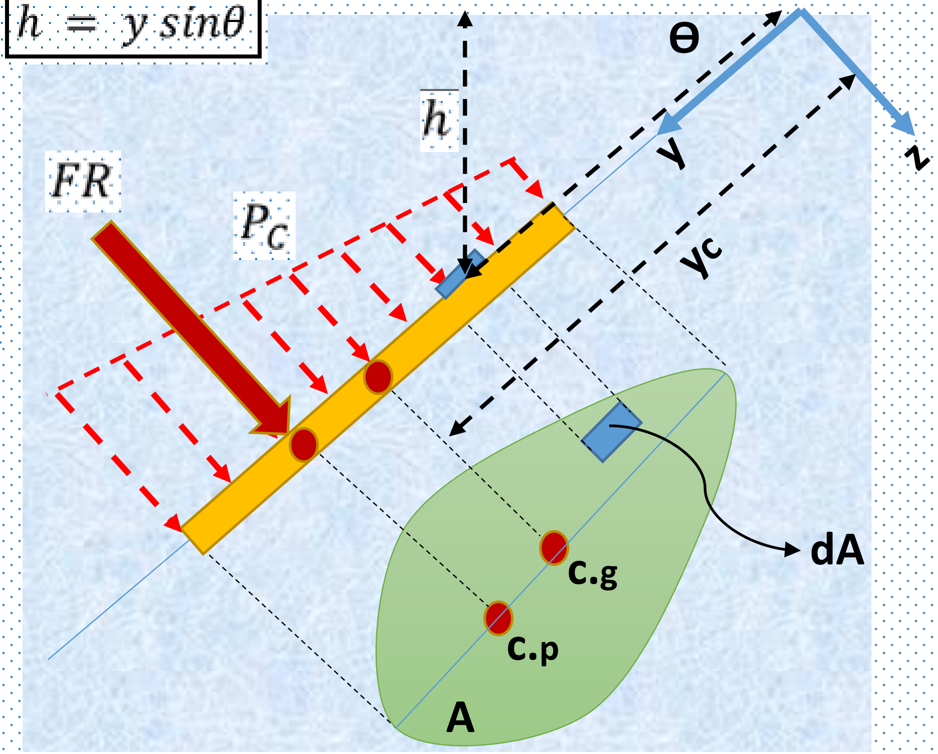
The resultant hydrostatic force FR acting on the surface is determined by $FR = \int P\,dA$

$$\therefore FR = \int_A P\ dA = \int_A (P_0 + \rho g y\sin\theta)\ dA = P_0 A + \rho g\sin\theta \int_A y\ dA$$

but, From the definition of the centroid equation

$$\because y_c = {}^1/_A \int_A y\ dA$$ *(first moment of inertia)*

$$\therefore FR = (P_0 A) + (\rho g\sin\theta)(y_c A) = (P0 + \rho g h_C)A = P_C A$$



The pressure P0 is usually atmospheric pressure, which can be ignored in most force calculations

The Hydrostatic force acts at a point namely; C.p ……..see next slide to determine the location of the FR

❷ The point of application (y_{cp})

Reduce the distributed pressure load into a Single force

$$y_{cp} F_R = \int y \, (P \, dA)$$

$$= \int_A y \, (P_0 + \rho g y \sin\theta) \, dA = P_0 \int y \, dA + \rho g \sin\theta \int_A y^2 \, dA$$

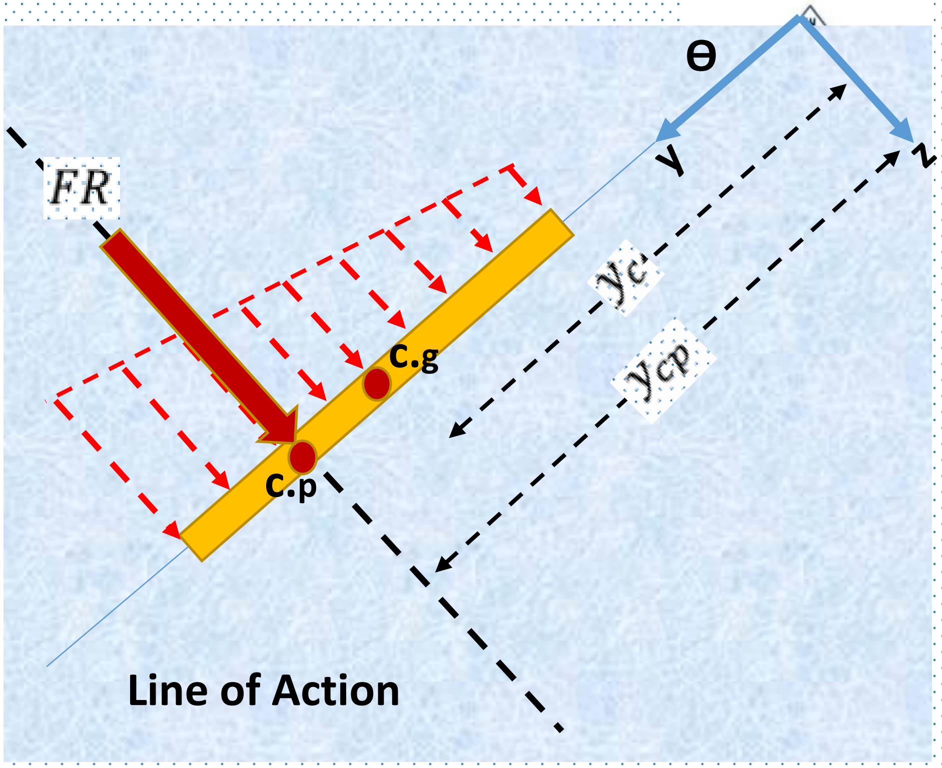
The second moment of inertia $\because I_{xx} = \int_A y^2 \, dA$

$$y_{cp} F_R = P_0 (y_c A) + \rho g \sin\theta \, I_{xx}$$

$$I_{xx} = I_{xx_{centroid}} + A(y_c^2) \qquad \textit{Parallel axis Therom}$$

$$y_{cp} = \frac{P_0 (y_c A) + \rho g \sin\theta \, (I_{xx_{centroid}} + A(y_c^2))}{(P_0 A) + (\rho g \sin\theta)(y_c A)}$$

Neglect the atmospheric pressure $\therefore y_{cp} = \frac{\rho g \sin\theta \, (I_{xx_{centroid}} + A(y_c^2))}{(\rho g \sin\theta)(y_c A)} = \frac{I_{xx_{centroid}} + A(y_c^2)}{(y_c A)} = y_c + \frac{I_{xx_{centroid}}}{(y_c A)}$



- ❑ **Conclusion:** The line of action of the resultant hydrostatic force, in general, **does not pass through the centroid of the surface**—It lies underneath where the pressure is higher. ***The point of intersection of the line of action of the resultant force and the surface is the center of pressure.***

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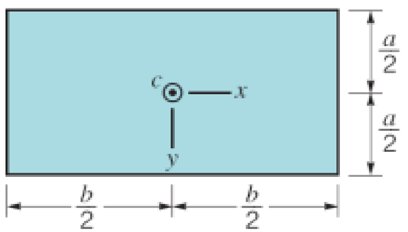
❖ Area Moment of Inertia Ixx



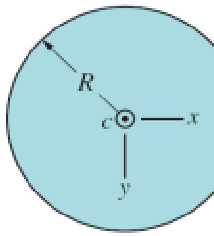
$A = ba$

$I_{xc} = \frac{1}{12} ba^3$

$I_{yc} = \frac{1}{12} ab^3$

$I_{xyc} = 0$

(*a*) Rectangle

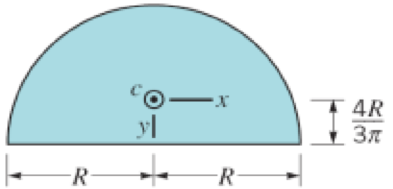


$A = \pi R^2$

$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$

$I_{xyc} = 0$

(*b*) Circle



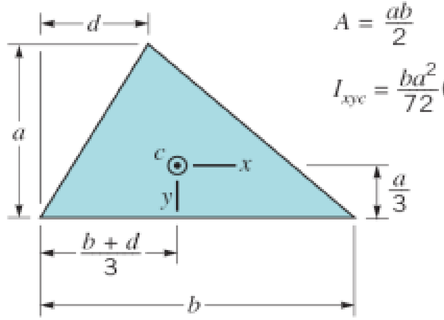
$A = \frac{\pi R^2}{2}$

$I_{xc} = 0.1098 R^4$

$I_{yc} = 0.3927 R^4$

$I_{xyc} = 0$

(*c*) Semicircle

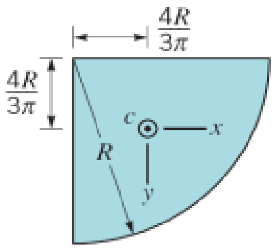


$A = \frac{ab}{2}$

$I_{xc} = \frac{ba^3}{36}$

$I_{xyc} = \frac{ba^2}{72}(b - 2d)$

(*d*) Triangle



$A = \frac{\pi R^2}{4}$

$I_{xc} = I_{yc} = 0.05488 R^4$

$I_{xyc} = -0.01647 R^4$

(*e*) Quarter circle

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EXAMPLE 3–8 Hydrostatic Force Acting on the Door of a Submerged Car

A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels (Fig. 3–35). The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.

The car door is considered as a vertical surface with an area of 1.2 m²

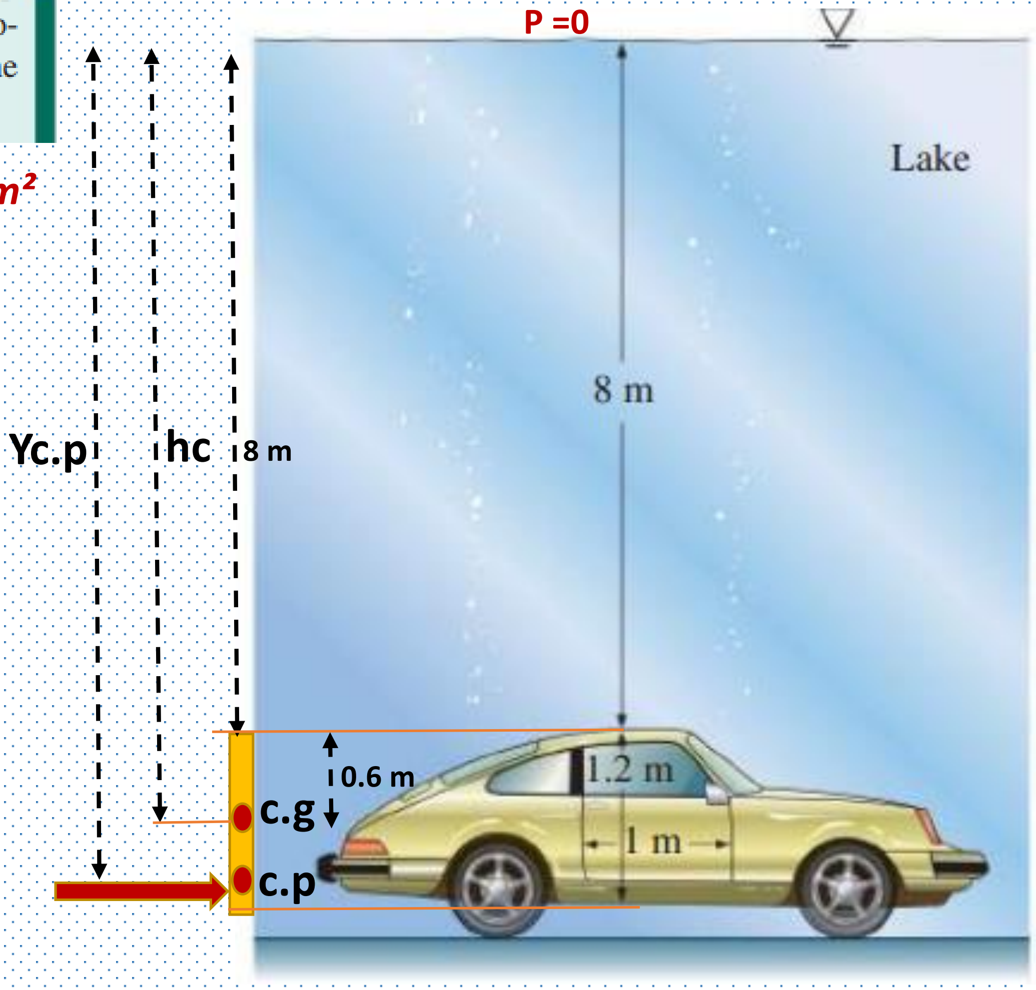
FH= (ɣ hc) A= 9810*(8+0.6)*(1.2*1) = 101239.2 N

The pressure center is directly under the midpoint of the door, and its distance from the surface of the lake is determined from:

Yc.p =hc+ (I/(hc.A)

Moment of Inertia for rectangular area I(xx) = $(h)^3(b)/12$

Yc.p =hc+ (I/(hc.A)= (8.6)+((1.2)³(1)/12 /(8.6*1.2)= 8.613 m



❖ Discussion

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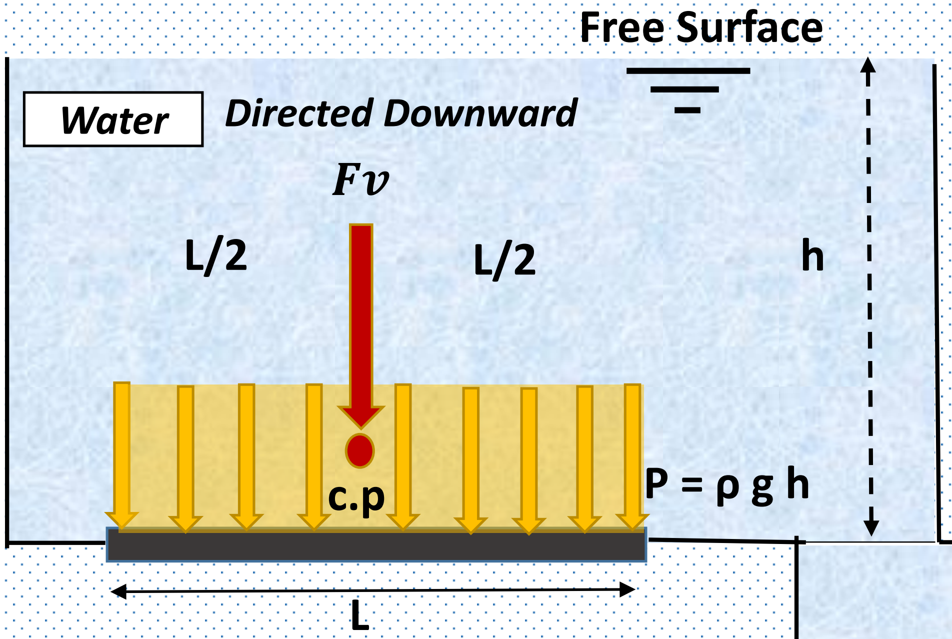
- A strong person can lift 100 kg, which is a weight of 981 N or about 1 kN.
- Also, the person can apply the force at a point farthest from the hinges (1 m farther) for maximum effect and generate a moment of 1 kN·m.
- The resultant hydrostatic force acts under the midpoint of the door, and thus a distance of 0.5 m from the hinges. This generates a moment of 50.6 kN·m, which is about 50 times the moment the driver can possibly generate.

❖ **Therefore, it is impossible for the driver to open the door of the car.**

✓ **The driver's best bet is to let some water in (by rolling the window down a little, for example) and to keep his or her head close to the ceiling. The driver should be able to open the door shortly before the car is filled with water since at that point the pressures on both sides of the door are nearly the same and opening the door in water is almost as easy as opening it in air.**

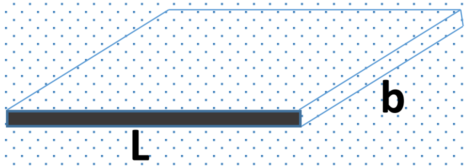
1 Hydrostatic Force on Horizontal Surface

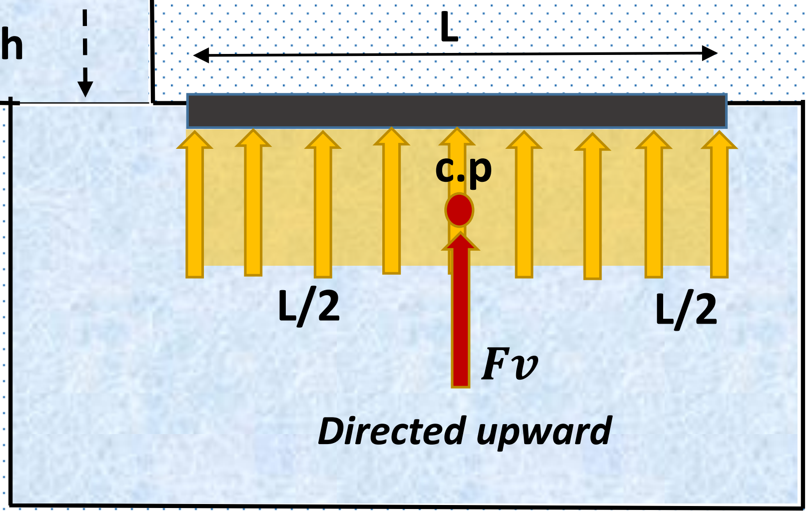
- ❖ water above Surface
- ❖ water below Surface



- Width b=1 if not given

Surface Area A= (L)(b)





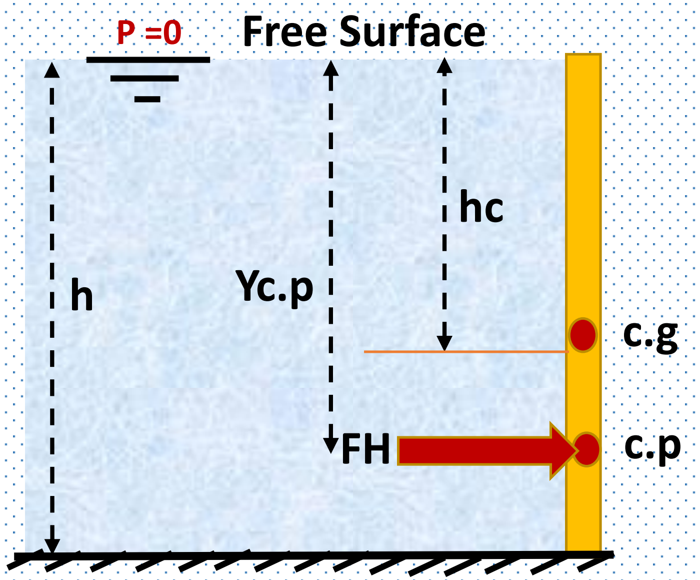
$$Fv = P.A = (\rho g h)A = (\gamma h)A = \gamma V$$

Point of application: (Fv) Acts at the middle of the horizontal surface

- h: Vertical distance from the surface to the free surface of the water
- A : Surface area
- γ : Specific weight of water = 9810 N/m³
- V: Volume of water over the gate to the free surface

2 Hydrostatic Force on vertical Surface

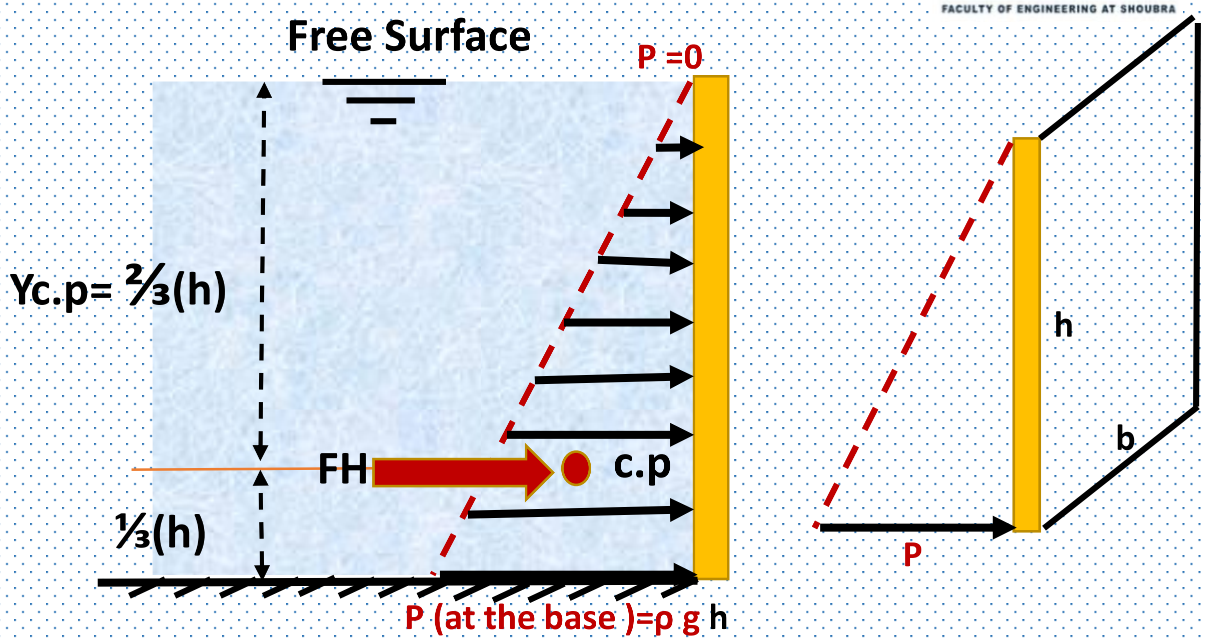
- Exact Method



$$F_H = P.A = (\rho g h_c)A = (\gamma hc)A$$

$$y_{c.p} = h_c + \frac{I}{h_c.A} = \frac{2}{3}h$$

- Pressure Diagram Method



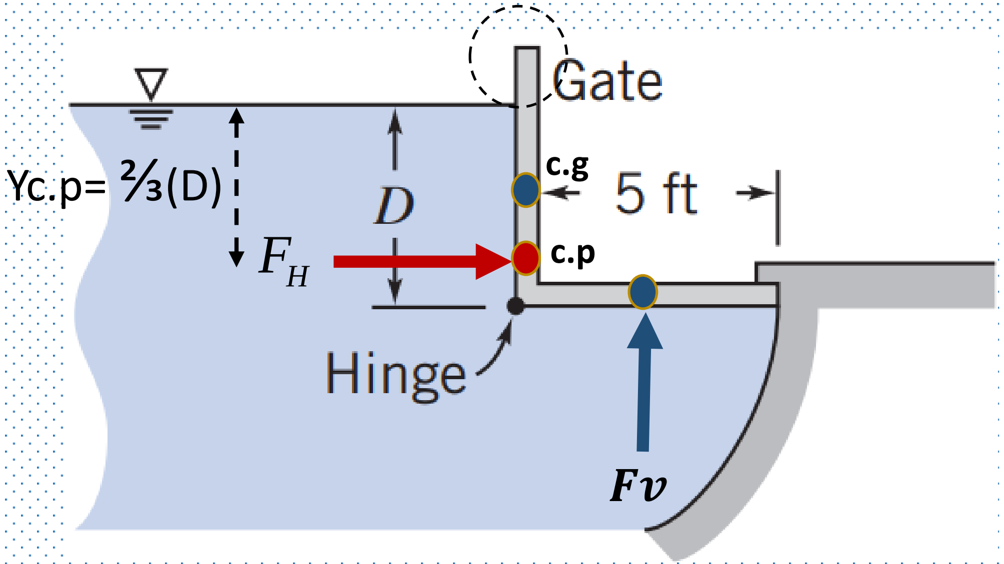
$$F_H = Volume\ of\ the\ pressure\ prism = \left(\frac{1}{2}\right) p.A$$

Located at the centroid of the pressure distribution C.p at a distance (2/3)(h) from the free surface

❖ Homework Assignment

As water rises on the left side of the rectangular gate, the gate will open automatically. At what depth above the hinge will this occur? Neglect the mass of the gate.

1- Find the hydrostatic force on the vertical surface as a function of D as magnitude and direction



2- calculate the hydrostatic force on the horizontal surface as a function of D as a magnitude and direction

3- The moment about the hinge equal to zero

❖ **Specific weight of water γ= 62.4 1b/ft³ for BG units**

3 Hydrostatic Force on vertical Surface

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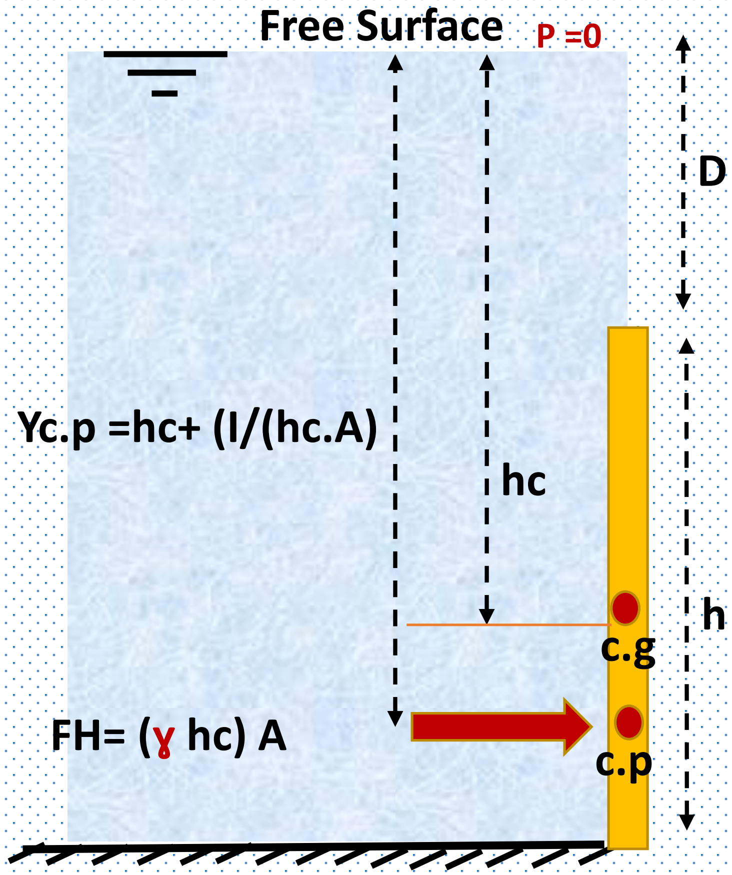
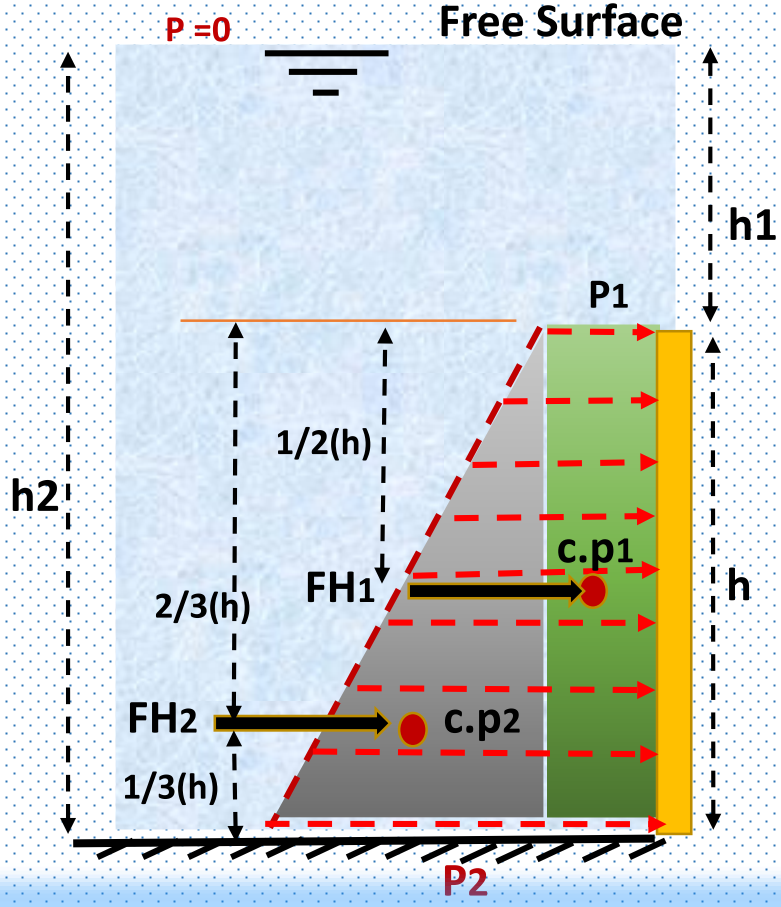
❑ Exact Method



❑ Pressure Diagram Method



$$P_1 = \rho g\ h_1$$

$$P_2 = \rho g\ h_2$$

$$F_{H1} = P_1.A$$

$$F_{H2} = (\tfrac{1}{2})(p_2 - p_1).(A)$$

$$F_{H(Total)} = F_{H1} + F_{H2}$$

Location of each force are applied at the centroid $c.p_1$ and $c.p_2$

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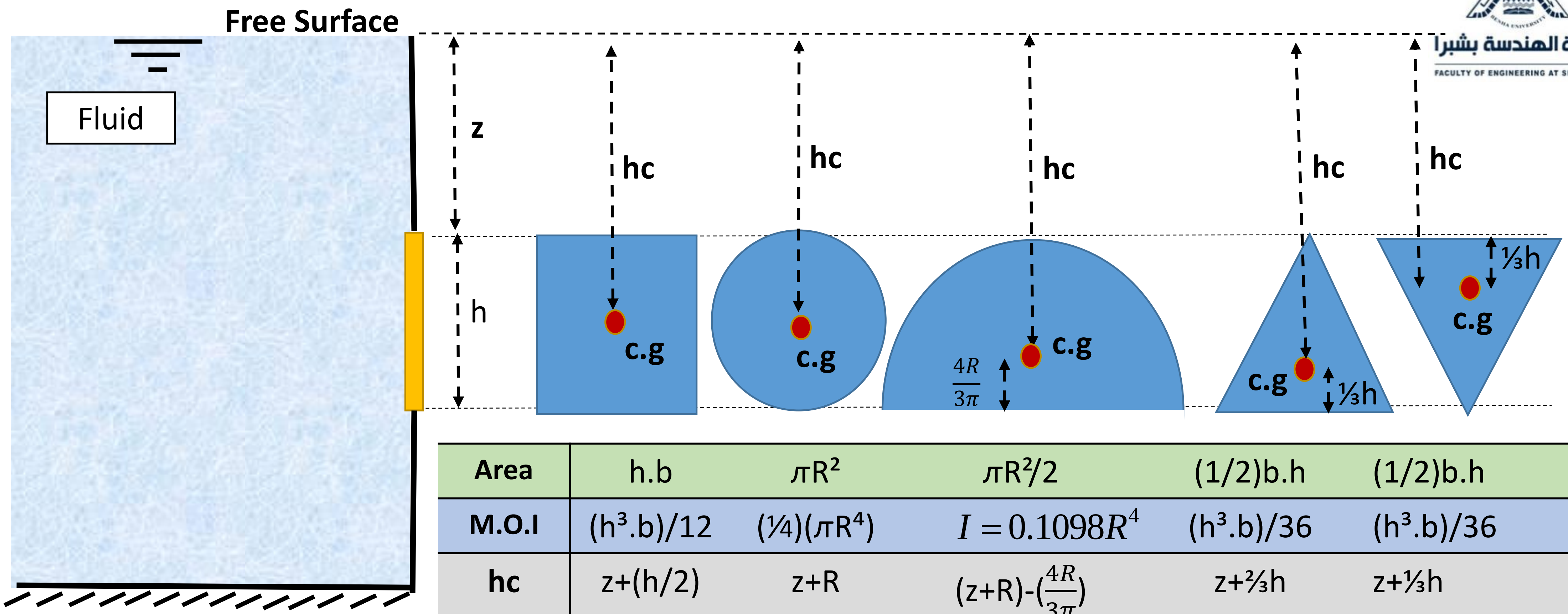
❖ Keep in mind the different shapes of the gate surface.

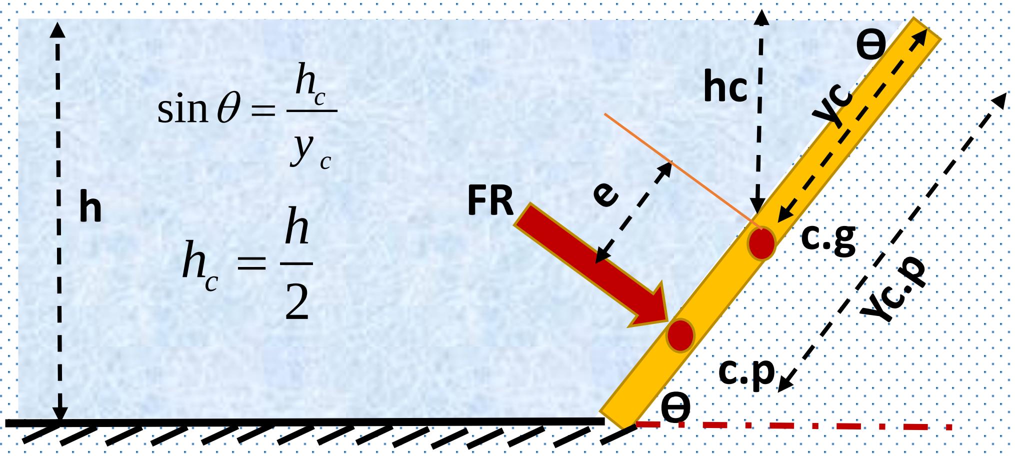


Area	h.b	πR^2	$\pi R^2/2$	(1/2)b.h	(1/2)b.h
M.O.I	$(h^3.b)/12$	$(1/4)(\pi R^4)$	$I = 0.1098R^4$	$(h^3.b)/36$	$(h^3.b)/36$
hc	z+(h/2)	z+R	$(z+R)-(\frac{4R}{3\pi})$	z+⅔h	z+⅓h

- **(hc) is the distance from the centroid of the gate to the free surface**
- **According to the shape of the gate , select the suitable area and moment of inertia about x- axsis**

4 Hydrostatic Force on Inclined Surface

❑ Exact Method

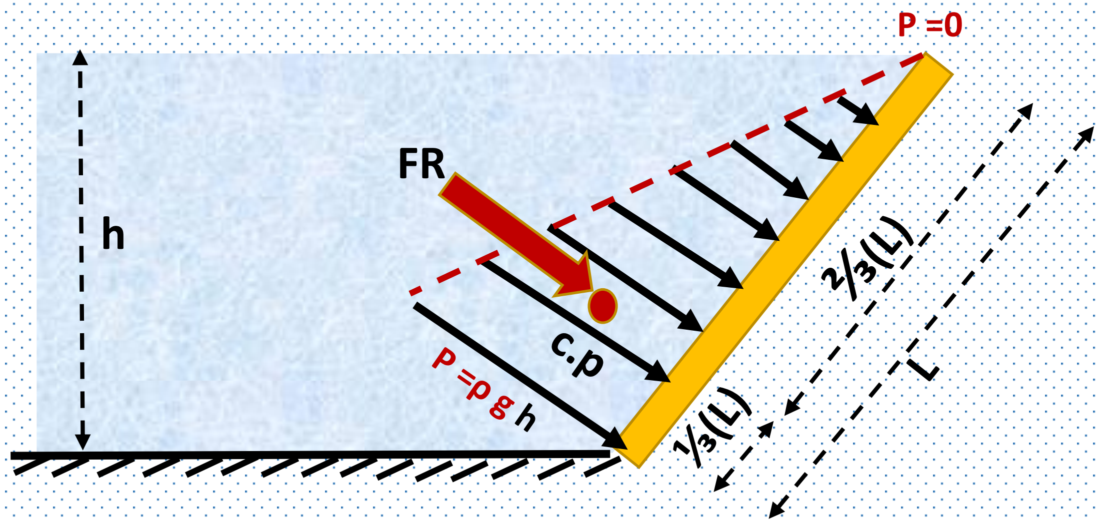


$$F_R = (\rho g h_c)A = (\gamma hc)A = (\gamma\, yc.\sin\theta)A$$

$$y_{c.p} = y_c + \frac{I}{y_c.A}$$

$$e = y_{c.p} - y_c = \frac{I}{y_c.A}$$

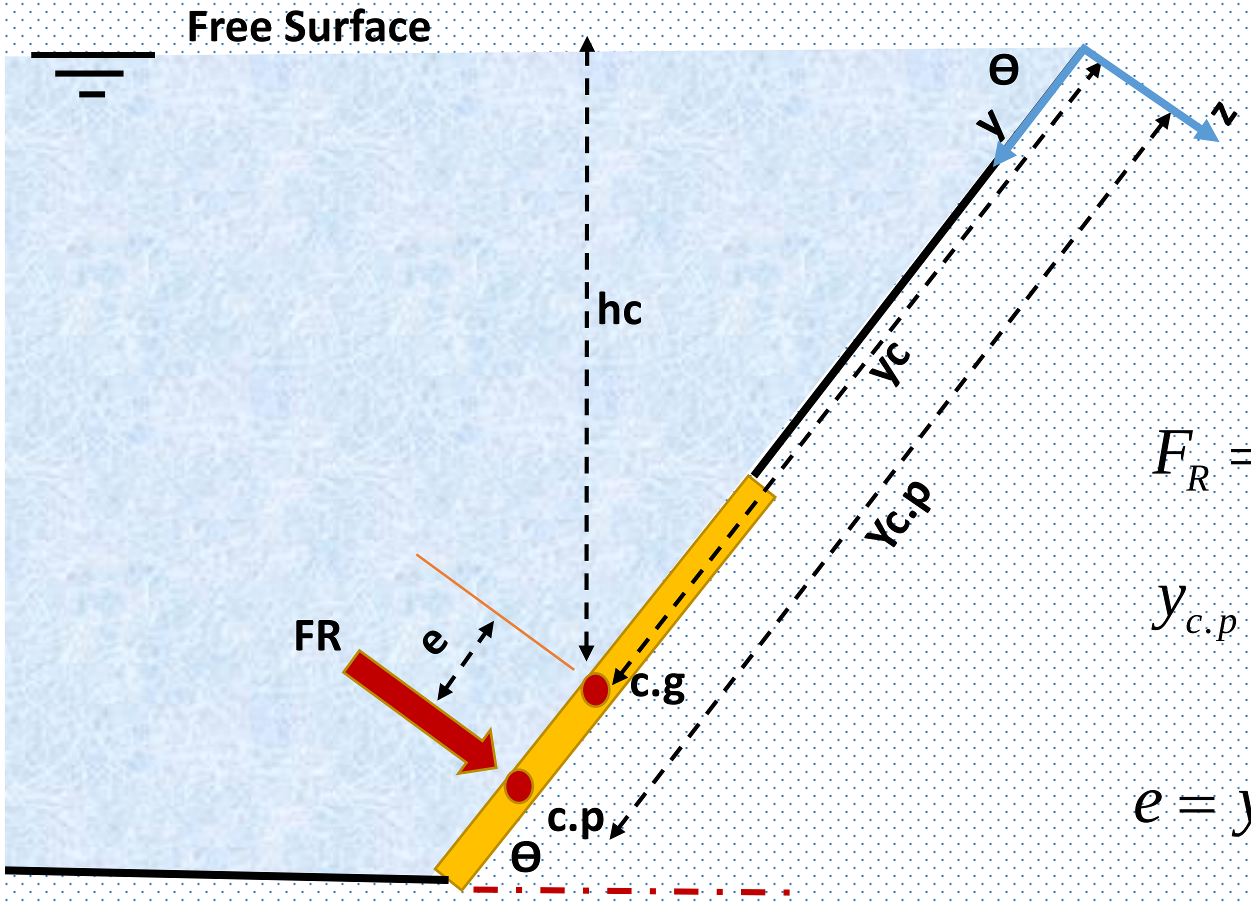
❑ Pressure Diagram Method



$$F_R = \text{Volume of the pressure prism} = \left(\frac{1}{2}\right)p.A$$

$$A = L.b \qquad L = \frac{h}{\sin\theta}$$

5 Hydrostatics Force on Inclined Surface



- Exact Method

$$\sin\theta = \frac{h_c}{y_c}$$

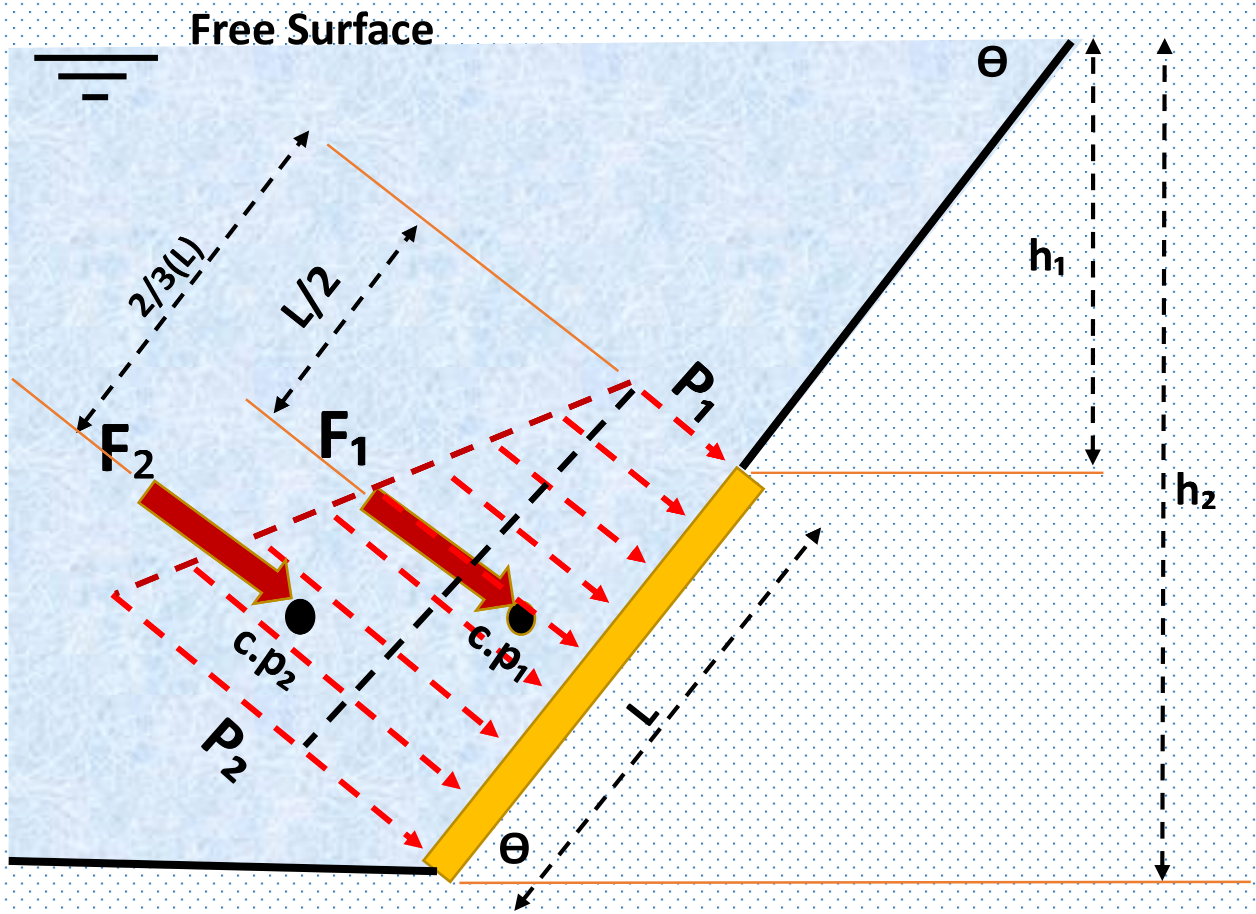
$$F_R = (\rho g h_c)A = (\gamma hc)A = (\gamma yc.\sin\theta)A$$

$$y_{c.p} = y_c + \frac{I}{y_c.A}$$

$$e = y_{c.p} - y_c = \frac{I}{y_c.A}$$

5 Hydrostatics Force on Inclined Surface

- Pressure Diagram Method



$$P_1 = \rho g\, h_1$$

$$P_2 = \rho g\, h_2$$

$$F_{H1} = P_1 . A$$

$$F_{H2} = (\tfrac{1}{2})(p_2 - p_1).(A)$$

$$F_{H(Total)} = F_{H1} + F_{H2}$$

3.66 The gate shown is hinged at H. The gate is 3 m wide normal to the plane of the diagram. Calculate the force required at A to hold the gate closed.

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❑ Pressure Diagram Method

$$P_1 = \gamma_w . h_1 = 9810*(1.5) = 14715\ N/m^2$$

$$P_2 = \gamma_w . h_2 = 9810*(1.5+3\sin 30) = 28430\ N/m^2$$

$$F_1 = P_1 * A = 14715*(3*3) = 132435\ N$$

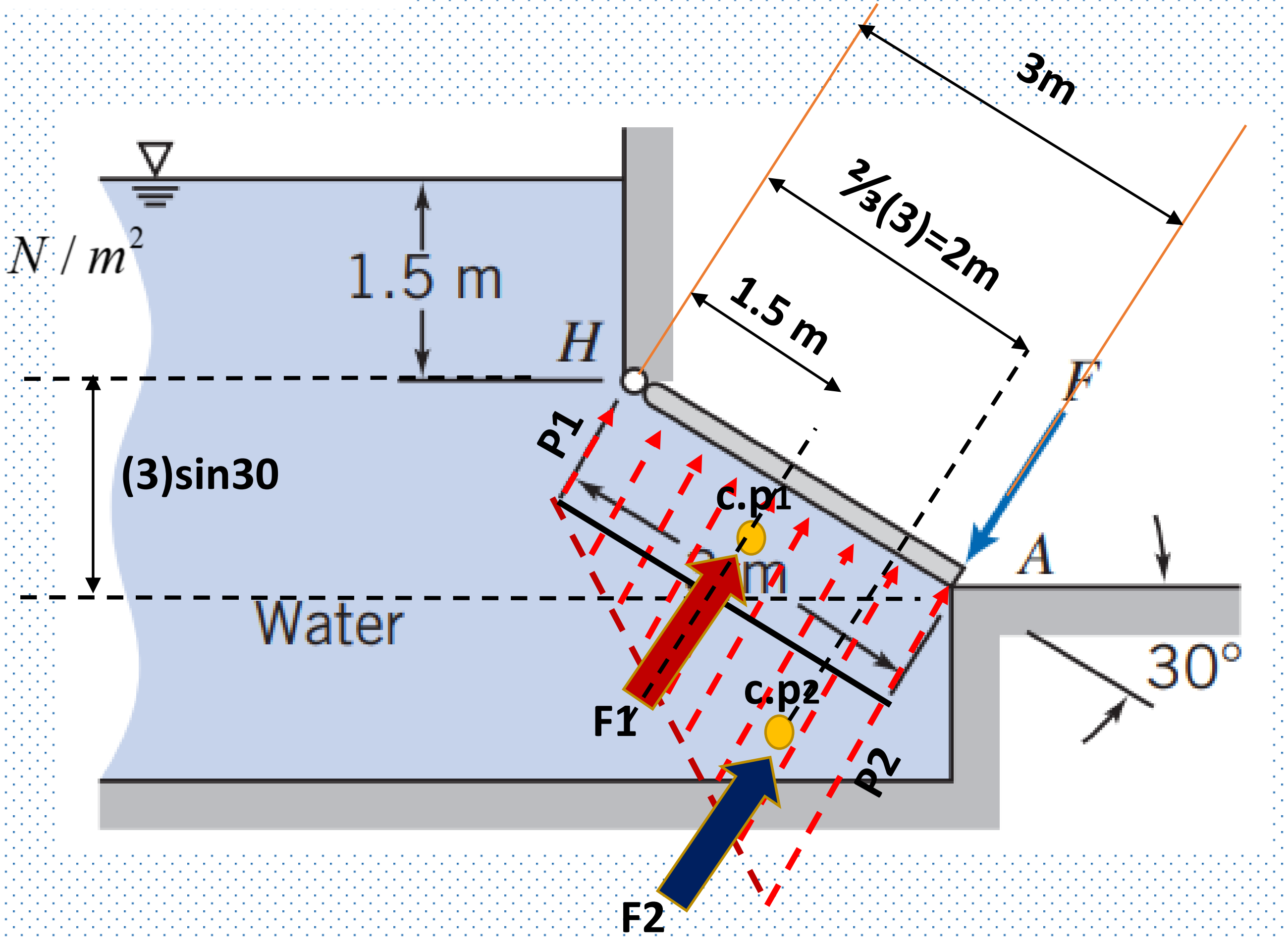
$$F_2 = \frac{1}{2}(P_2 - P_1)*A = \frac{1}{2}(28430-14715)*9 = 61717.5\ N$$

$$\sum M_{Hinge} = 0$$

$$F_1(1.5) + F_2(2) - F(3) = 0$$

$$132435\ (1.5) + 61717.5(2) - F(3) = 0$$

$$\therefore F = 107362.5\ N = 107.36\ kN$$



❑ Exact Method

$$F_R = \gamma_w . h_c . A$$

$$= 9810 * (1.5 + 1.5 \sin 30) * (9) = 198652.5\ N$$

$$y_c = \frac{h_c}{\sin 30} = \frac{1.5 + 1.5 \sin 30}{\sin 30} = 4.5\ m$$

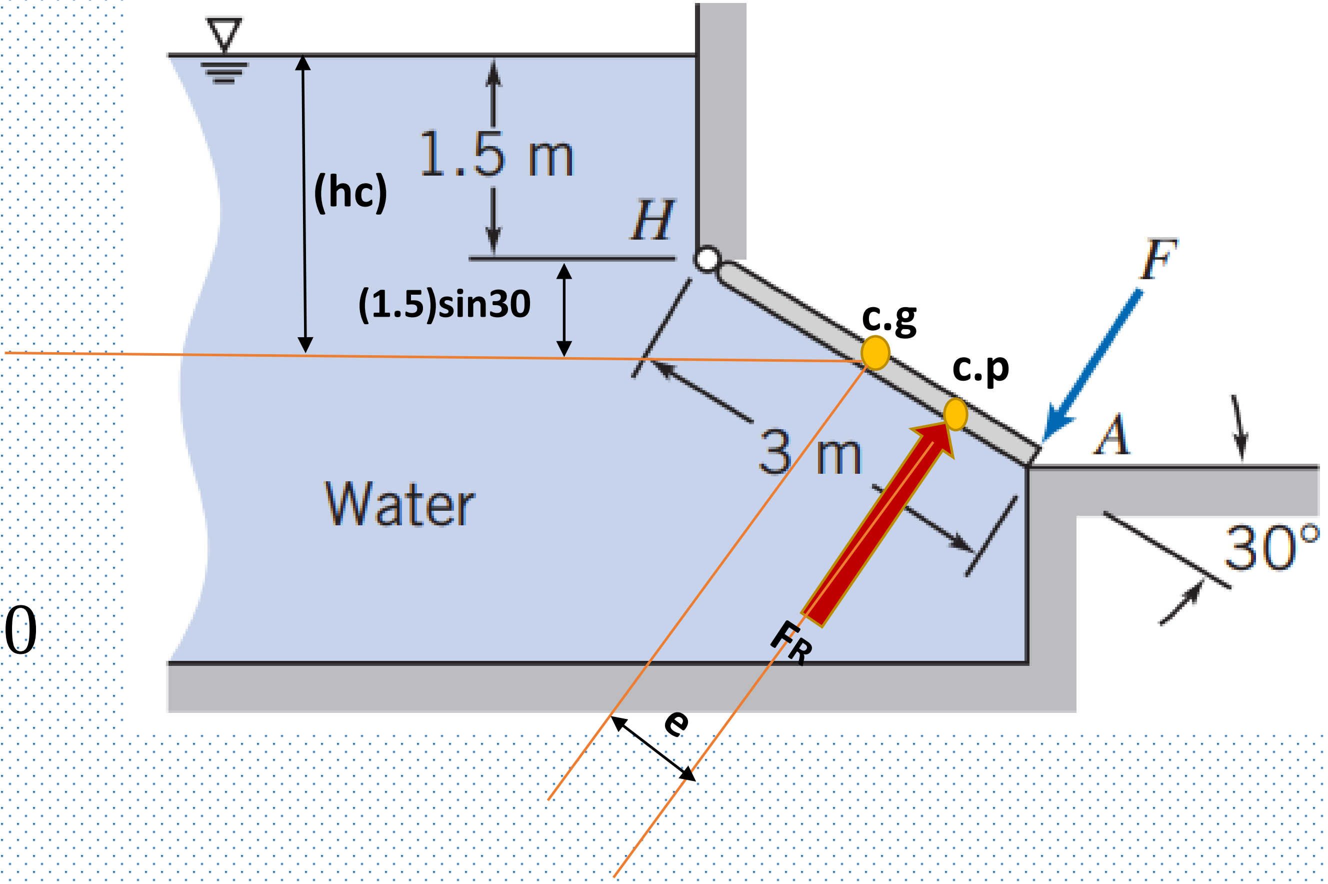
$$Ixx = \frac{(3)(3)^3}{12} = 6.75\ m^4$$

$$\therefore e = \frac{Ixx}{y_c . A} = \frac{6.75}{4.5 * 9} = 0.1666\ m$$

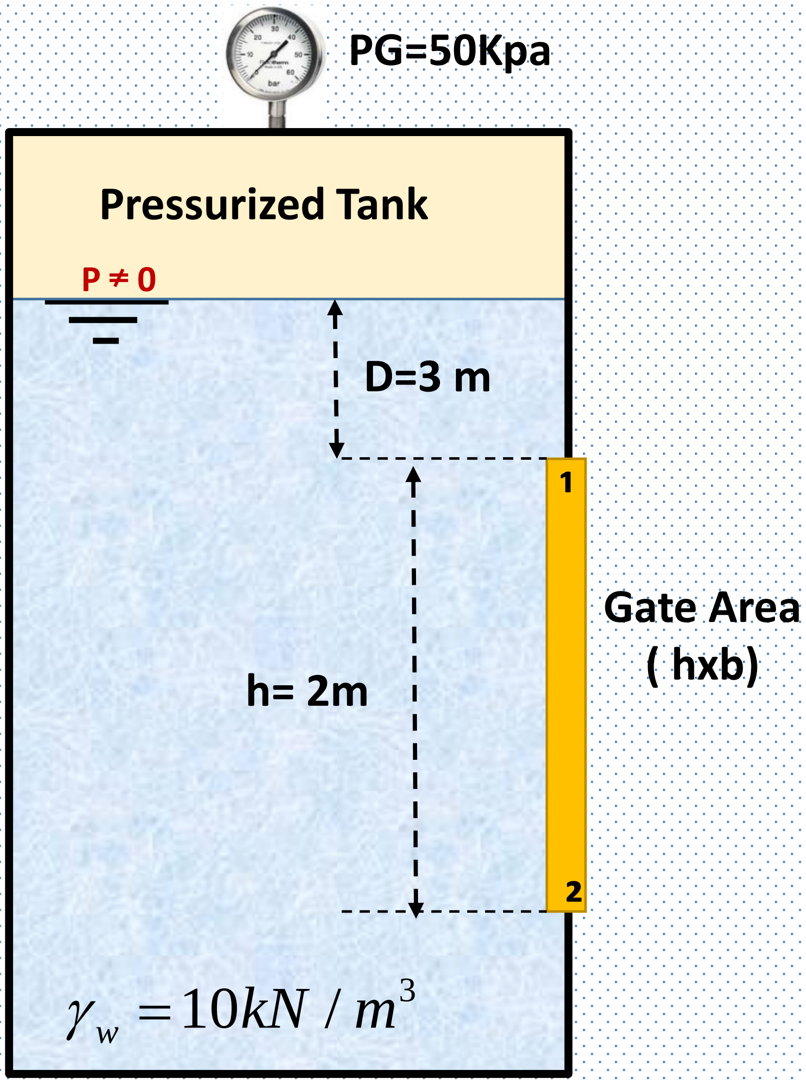
$$\sum M_{Hinge} = 0 \qquad F_R(1.5 + e) - F(3) = 0$$

$$198652.5(1.5 + 0.1666) - F(3) = 0$$

$$F = 110362.5\ N = 110.36\ kN$$



❖ (Homework Assignment) Hydrostatic Force on a surface contained in pressurized tank



❑ Exact Method

- Find the equivalent head to the pressure indicated by the pressure gage ($h_{eqv} = P/\gamma w$)
- Add the equivalent head to the liquid originally that exists in the tank.
- Find the force F_H
- Find the location $Y_{c.p}$

❑ Pressure Diagram Method

- Find the pressure at point 1 and 2
- $F_{H_1} = P_1 . A$
- $F_{H_2} = (1/2)(p_2 - p_1).(A)$
- Specify the location of each force on the surface
- $F_{H\ (Total)} = F_{H_1} + F_{H_2}$
- Specify the location of the total force on the gate from the original water surface in the tank.

6 Hydrostatic force on curved surface

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$$F_V = F_{V1} + F_{V2} = (\rho g V_1) + (\rho g V_2) = \gamma (V_1 + V_2) = 1092834\ N$$

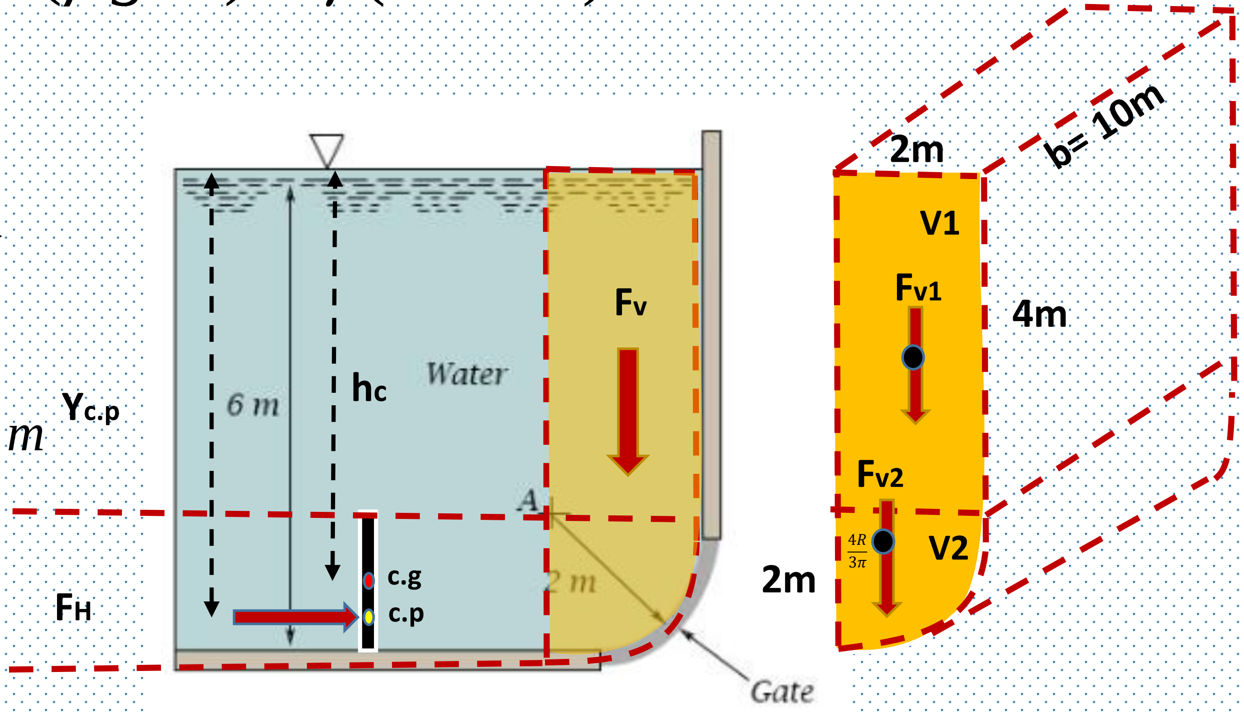
$$F_H = (\rho g h_c) A = (\gamma h_c) A$$

$$= 9810 * 5 * (2 * 10) = 981000\ N$$

$$y_{c.p} = h_c + \frac{I}{h_c . A} = 5 + \frac{\frac{(10)*(2)^3}{12}}{(5)*(2*10)} = 5.066\ m$$

$$F_R = \sqrt{(F_V)^2 + (F_H)^2}$$

$$= \sqrt{(1092834)^2 + (981000)^2} = 1468.55\ kN$$



$$V_1 = 2x4x10 = 80 m^3$$

$$V_2 = \left(\frac{1}{4}\right)(\pi R^2)(b) = \left(\frac{1}{4}\right)(\pi 2^2)(10) = 31.4\ m^3$$

The homogeneous gate shown consists of one **quarter of a circular** cylinder and is used to maintain a water depth of **4 m**. That is, when the water depth exceeds **4 m**, the gate opens slightly and lets the water flow under it. ***Determine the weight of the gate per meter of length***

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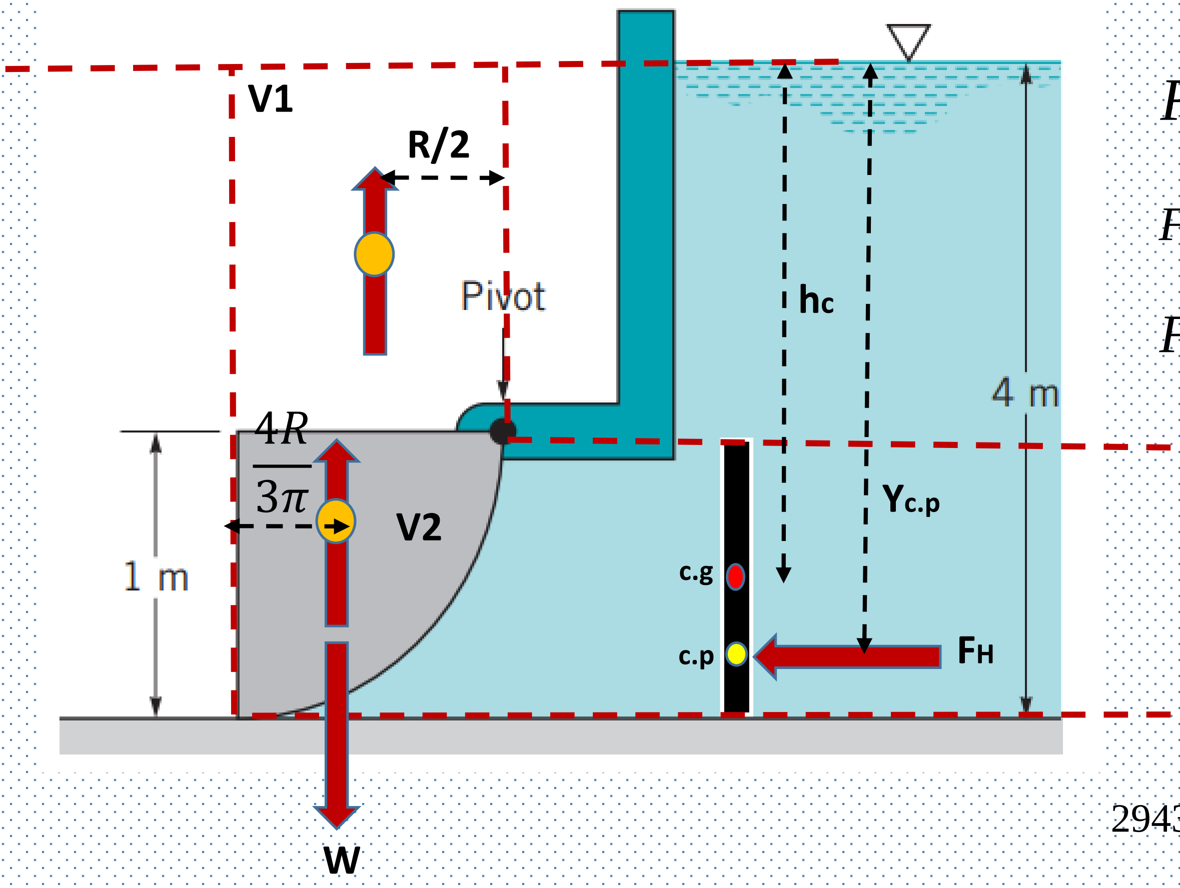

$$Fv_1 = \rho g V_1 = 1000*9.81*(3*1*1) = 29430\ N$$

$$Fv_2 = \rho g V_2 = 1000*9.81*(\frac{1}{4}[\pi(1)^2*1]) = 7700.85\ N$$

$$F_H = \rho g h_c A = 1000*9.81*3.5*(1*1) = 34335\ N$$

$$y_{c.p} = h_c + \frac{I}{h_c.A} = 3.5 + \frac{\frac{(1)*(1)^3}{12}}{(3.5)*(1*1)} = 3.5238\ m$$

$$\sum M_{Hinge} = 0$$

$$Fv_1*(0.5) + Fv_2*(1-\frac{4(1)}{3\pi}) + F_H*(y_{c.p}-3) - W*(1-\frac{4(1)}{3\pi}) = 0$$

$$29430*(0.5) + 7700.85*(1-\frac{4(1)}{3\pi}) + 34335*(3.5238-3) - W*(1-\frac{4(1)}{3\pi}) = 0$$

$$W = 64533.69\ N = 64.5\ kN$$

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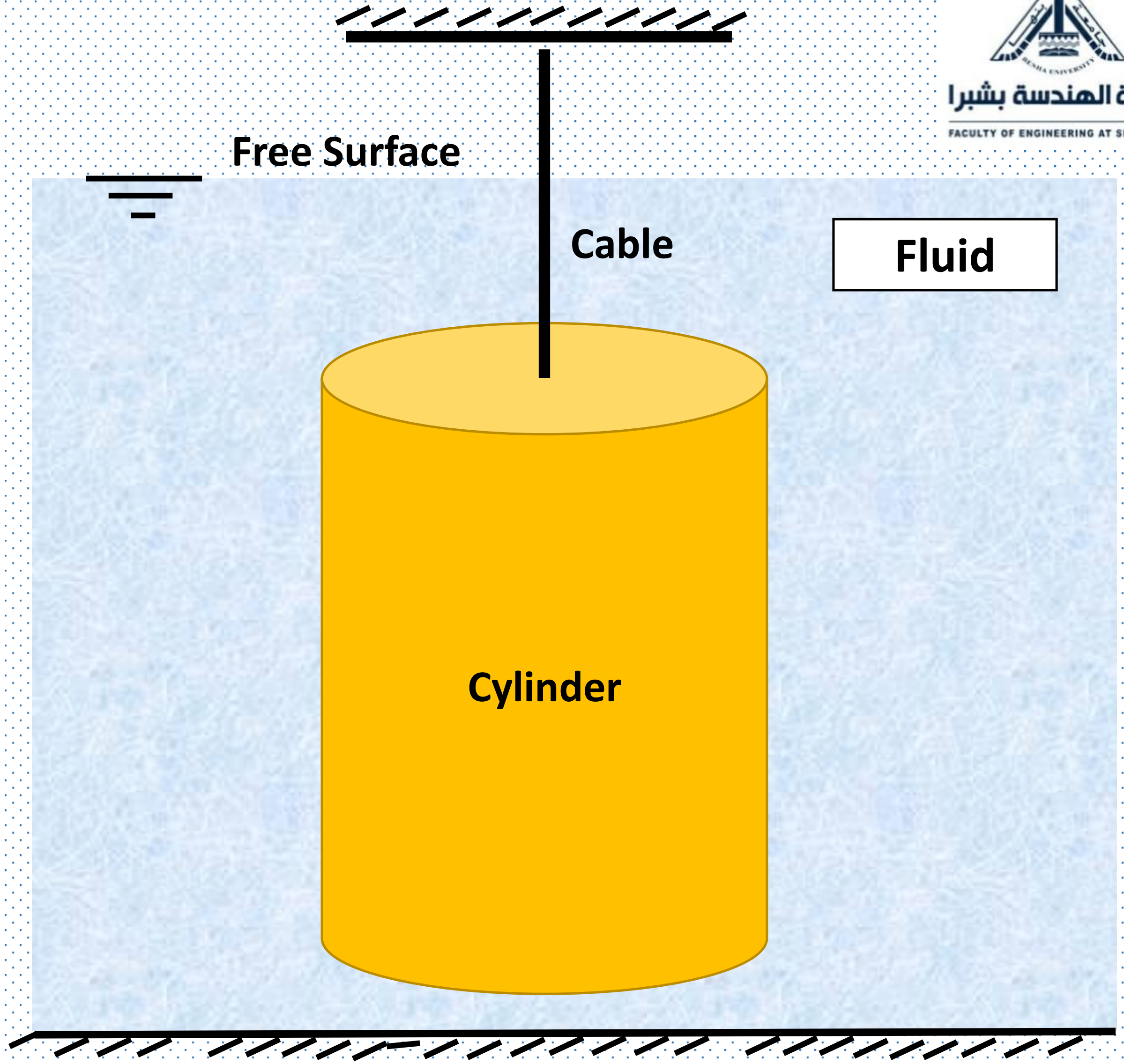
❖Quiz

Find the net horizontal force acting on the shown cylinder?



❖ Lab Experiment no. 2
Hydrostatic Force and Center of Pressure

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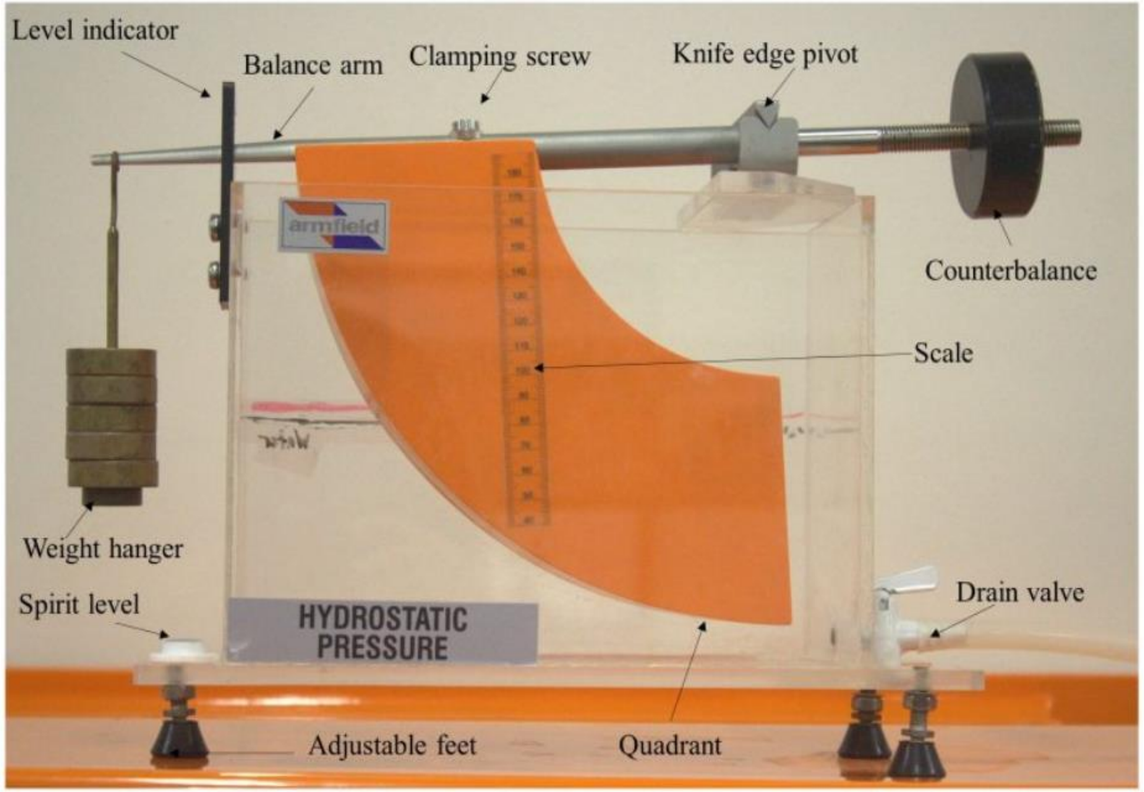
- This experiment is designed to help you understand how to locate the center of pressure and compute the hydrostatic force acting on a submerged surface

- To determine experimentally the resultant hydrostatic force (total force) applied on a submerged surface and to determine the experimental and the theoretical center of pressure



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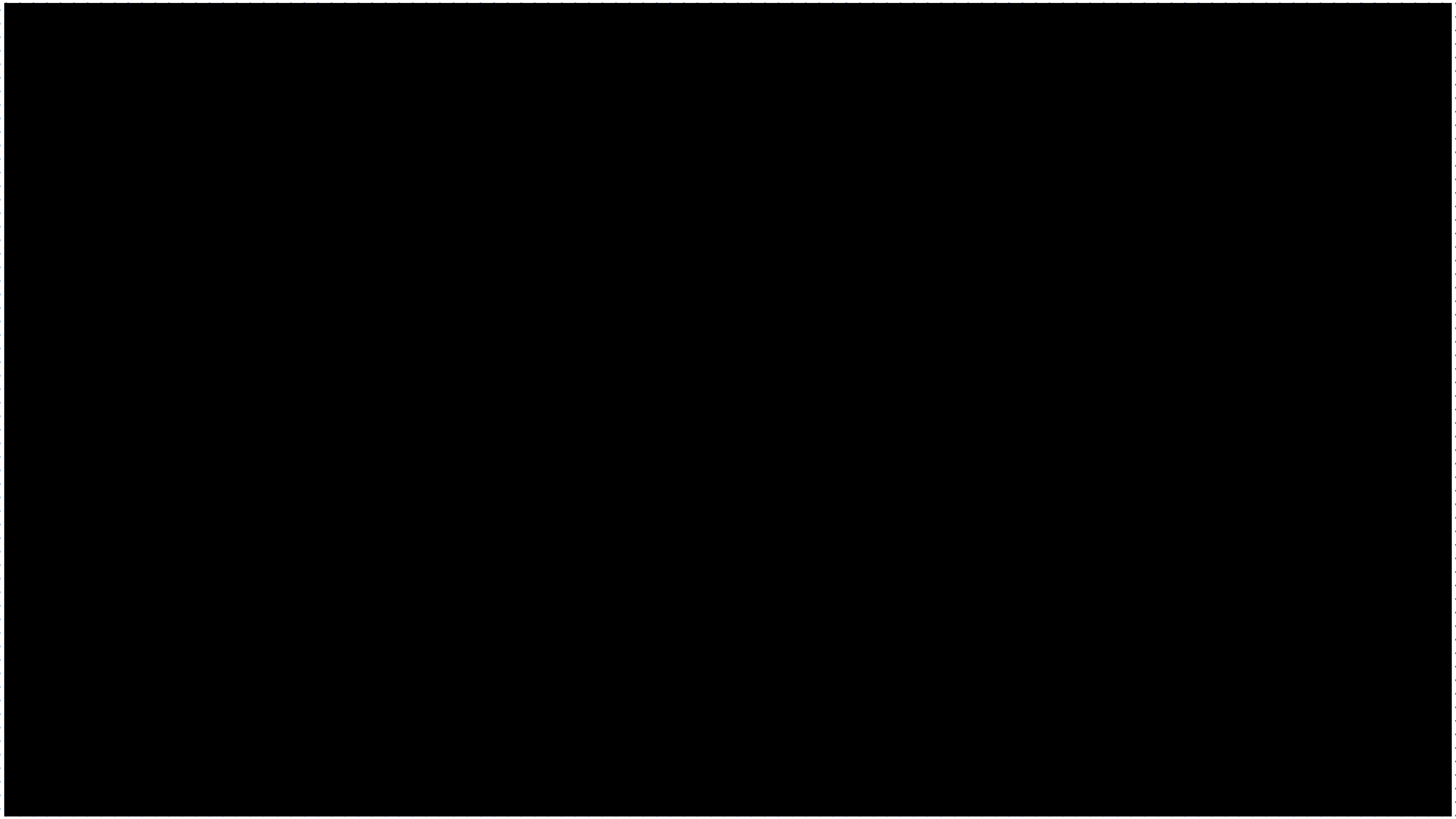

https://www.youtube.com/watch?v=LfqadPBKim8

Thank you